



TEST IN MATHEMATICS

2015

I n s t r u c t i o n s

You are given the test booklet and the answer sheet.

Read the descriptions of the task types carefully.

The test consists of 40 tasks. In the end of each task the maximum score for the task is indicated.

Tasks 1-30 are multiple choice tasks with only one correct answer. Maximum score for each of these tasks is 1.

Tasks 31-40 are open-ended, which means that you have to write the answers in the space specially provided for this. **In your answer the way for solving the problem must be stated clearly.**

All the answers must be transferred onto the mark sheet. Fill in the answer sheet attentively! Write clearly, use the space given!

Take into consideration that **only the answer sheet will be marked.**

The answers which are written (or circled) in the test booklet will not be marked! Test booklet can only be used as a working (draft) sheet only!

Do not write your name or surname. The answer sheet which has an applicant's name and/or surname, or any other means of personal identification (e.g. a nickname) will not be marked!

You are given 3 hours and 30 minutes for the test.

We wish you success!

Problem 1**1 point**

$$0,99 : 1,1 + 0,1 =$$

a) $\frac{1}{11}$

b) 1

c) 1,1

d) 9,1

Problem 2**1 point**

A number a gives a remainder 2 when divided by 8. What remainder is left by the number $27 + a$ when divided by 8?

a) 7

b) 5

c) 3

d) 2

Problem 3**1 point**

As a result of cheaper oil, the price per liter of gasoline dropped by 20% and constituted 1,68 Lari. What was the price per liter of gasoline before the price dropped?

a) 2,016 Lari

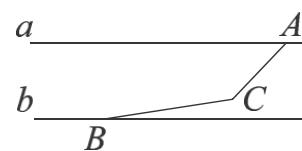
b) 2,05 Lari

c) 2,1 Lari

d) 2,25 Lari

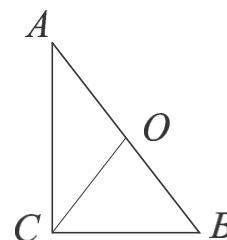
Problem 4**1 point**

Based on the figure, find the angle ACB , if the segments AC and BC make angles 58° and 12° with parallel straight lines a and b , respectively.

a) 146° b) 120° c) 46° d) 134°

Problem 5**1 point**

In the right triangle ABC ($\angle C = 90^\circ$), the segment CO is a median. Find the length of the leg AC , if $CO = CB = 5$.

a) $5\sqrt{3}$

b) 6

c) 10

d) $\sqrt{15}$

Problem 6**1 point**

At some bus stop bus №11 stops every 12 minutes, and the bus №17 - every 18 minutes. At some point in time, these buses simultaneously stopped at this stop. Find the smallest time interval, after which these buses will meet at this bus stop again.

a) After 30 minutes

b) After 36 minutes

c) After 96 minutes

d) After 216 minutes.

Problem 7**1 point**Find n if $\frac{4,5}{\sqrt{n}} = \frac{9}{50}$.

a) 5

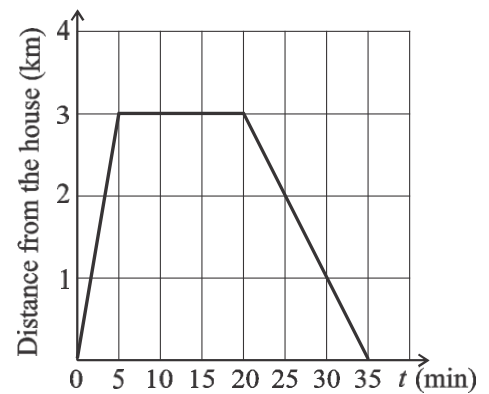
b) 225

c) 25

d) 625

Problem 8**1 point**

The cyclist left the house and rode down the slope to a store. After purchasing goods he returned back home by the same road. The graph shows the dependence on time of the distance from the cyclist to the house. Using the graph, determine the velocity of the cyclist on the rise (assuming that the cyclist was moving along a straight line).



a) 10 km/h

b) 15 km/h

c) 9 km/h

d) 12 km/h

Problem 9**1 point**If $a - b = 1$ then $a^3 - b^3 =$

a) 1

b) $1 + 3b + 3b^2$ c) $1 - 3b + 3b^2$ d) $1 + 2b$

Problem 10**1 point**Linear equation $ax = b$ has no solution when

- a) $a = 0$ and $b = 0$ b) $a \neq 0$ c) $b = 0$ d) $a = 0$ and $b \neq 0$

Problem 11**1 point**In a Cartesian coordinate system find the equation of the line passing through the points $(3; 2)$ and $(-1; -2)$.

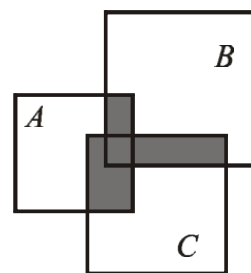
- a) $y = -x + 5$ b) $y = \frac{x}{2} - \frac{3}{2}$ c) $y = x - 1$ d) $y = x^2 - x - 4$

Problem 12**1 point**Brother is 5 years older than his sister. Find the sum of the ages of these siblings, if four years ago the ratio of their ages was $3:2$.

- a) 23 b) 27 c) 33 d) 36

Problem 13**1 point**

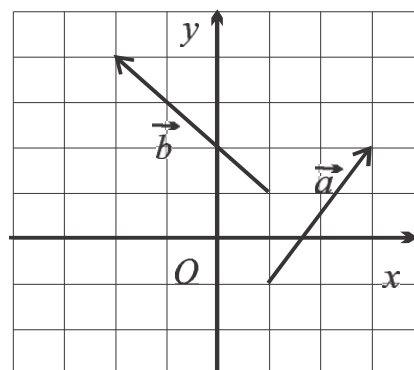
In the Venn diagram shown in the picture, the sets A , B and C are depicted as squares. Which of the following sets stated below is given by the shaded part of this diagram?



- a) $A \cup (B \cap C)$
- b) $(A \cap C) \cup (A \cap B) \cup (B \cap C)$
- c) $A \cap B \cap C$
- d) $A \cup B \cup C$

Problem 14**1 point**

On a squared paper sheet, each cell of which is a unit square, vectors \vec{a} and \vec{b} are depicted so, that the beginnings and ends of each vector lie on the vertices of the cells (see the figure). Based on the figure, determine the coordinates of the vector $\vec{a} + \vec{b}$.



- a) $(-1; 6)$
- b) $(1; 3)$
- c) $(-1; 2)$
- d) $(1; 6)$

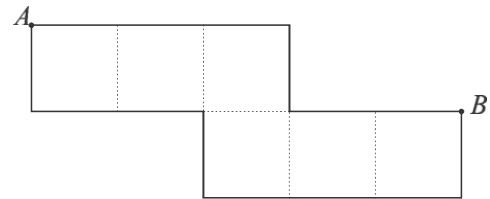
Problem 15**1 point**

In the first class there is one boy more and one girl less than in the second class. For each class a pie chart is built representing numbers of boys and girls. Find how many students are in the first class, if the central angle of the sector corresponding to the number of girls, on the second class' chart is greater by 30° than on the first class' chart.

- a) 30 b) 6 c) 24 d) 12

Problem 16**1 point**

The net of a cube is given in the figure. Find the distance between the vertices of the cube which corresponds to the points A and B on the net, if the volume of the cube is 1 cm^3 .



- a) 0 cm b) $\sqrt{3}$ cm c) 1 cm d) $\sqrt{2}$ cm

Problem 17**1 point**

The graphs of the functions $y = \log_2 x$ and $y = \log_2 \left(\frac{1}{x} \right)$ are constructed on a rectangular coordinate plane Oxy . Which of the transformations of the plane given below maps the graph of the first function on the graph of the second function?

- a) Symmetry with respect to the axis Oy .
b) Symmetry with respect to the axis Ox .
c) Symmetry with respect to the origin O .
d) Symmetry with respect to the line $y = x$.

Problem 18**1 point**

There are 8 red and 8 white balls in an urn. Randomly selected two balls are simultaneously drawn from the urn. Find the probability that the drawn balls will be of different colors.

- a) $\frac{7}{16}$ b) $\frac{1}{3}$ c) $\frac{1}{2}$ d) $\frac{8}{15}$

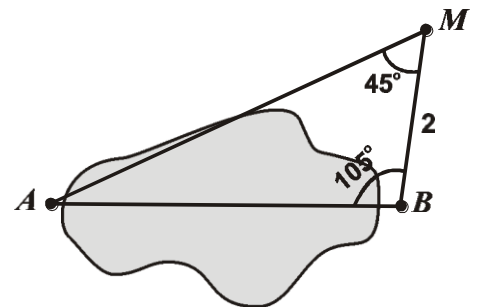
Problem 19**1 point**

If $f(x) = 2 - 3x$ and $g(x) = 2 - 5x$, then $f(g(x)) =$

- a) $15x - 8$ b) $4 - 8x$ c) $15x - 4$ d) $(2 - 3x) \cdot (2 - 5x)$

Problem 20**1 point**

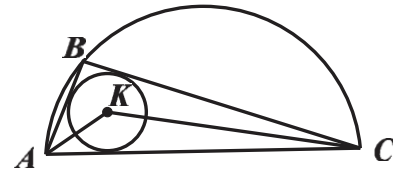
In order to measure the distance between the points A and B located on the shore of the lake, the distance from the point M to the point B and the angles AMB and ABM are measured. Find the distance AB if it is known that $MB = 2$ km, $\angle AMB = 45^\circ$ and $\angle ABM = 105^\circ$.



- a) $2\sqrt{2}$ km b) 3 km c) $2\sqrt{3}$ km d) 4 km

Problem 21**1 point**

A semicircle with a diameter AC is expressed on the figure. Point B lies on the arc of the semicircle, and the point K is the center of the circle inscribed in a triangle ABC . Which of the following statements is true?



- a) The measure of an angle AKC depends on the position of the point B on the arc of the semicircle.
- b) The measure of an angle AKC doesn't depend on the position of the point B on the arc of the semicircle, but it depends on the radius of the semicircle.
- c) The measure of an angle AKC doesn't depend on the position of the point B on the arc of the semicircle, but it depends on the radius of the circle inscribed in a triangle ABC .
- d) For any point B on the arc of the semicircle $\angle AKC = 135^\circ$.

Problem 22**1 point**

It is known that $\frac{a}{x+1} + \frac{b}{x-1}$ and $\frac{7x+10}{x^2-1}$ are identically equal expressions for some values of the parameters a and b . Find $a^2 - b^2$ for these values of the parameters a and b .

- a) -70
- b) -35
- c) 10
- d) 0

Problem 23**1 point**

Find the set of all values of a variable a for which the point $P(2a+3; 3a-2)$ lies in the fourth quadrant of the coordinate plane (but not on the coordinate axes).

a) $\left(\frac{2}{3}; \frac{3}{2}\right)$

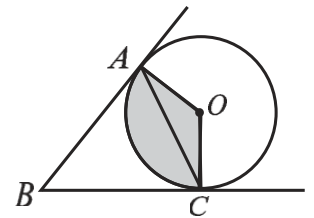
b) $\left(-\frac{3}{2}; \frac{2}{3}\right)$

c) $\left(\frac{2}{3}; +\infty\right)$

d) $\left(-\infty; \frac{2}{3}\right)$

Problem 24**1 point**

The sides of an angle ABC touches the circle at the points A and C (see the figure). Find the area of the shaded sector AOC in the figure, if $AC = 6$ and $\angle ABC = 60^\circ$.



a) 6π

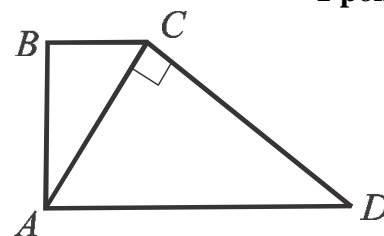
b) 4π

c) 3π

d) 9π

Problems 25**1 point**

In a rectangular trapezoid $ABCD$ the diagonal AC is perpendicular to the side CD (see the figure). Find the area of the trapezoid if $AC = 3$ and $AD = 5$.



- a) 6,9 b) $8\frac{1}{4}$ c) $8\frac{4}{25}$ d) $9\frac{4}{25}$

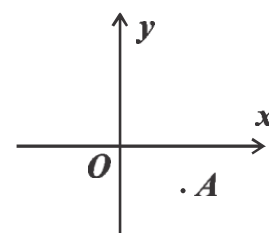
Problem 26**1 point**

Evaluate $\sin(\alpha - \beta)$, if $\cos \alpha = \frac{1}{3}$, $\cos \beta = \frac{1}{4}$, $\alpha \in (0; \pi)$ and $\beta \in \left(-\frac{\pi}{2}; 0\right)$.

- a) $\frac{\sqrt{23}}{12}$ b) $\frac{\sqrt{7}}{12}$ c) $\frac{\sqrt{15} + \sqrt{8}}{12}$ d) $\frac{\sqrt{15} - \sqrt{8}}{12}$

Problem 27**1 point**

The point $A(3; -2)$ is given in a Cartesian coordinate system Oxy . Find the coordinates of an image of A by successively applying the following two transformations: firstly, symmetry with respect to the line $y = x$ and then, translation by the vector $\vec{a} = (3; -5)$.



- a) $(-7; 6)$ b) $(0; -3)$ c) $(1; -2)$ d) $(6; -7)$

Problem 28**1 point**

In an arithmetic progression a_1, a_2, \dots consisting of positive numbers, the first term is three times greater than the difference of this progression. Find $\frac{a_{20}}{a_{10}}$.

- a) $\frac{11}{6}$ b) 2 c) $\frac{23}{13}$ d) 2,3

Problem 29**1 point**

What is the smallest positive period of the function $f(x) = |\sin 3x|$?

- a) $\frac{\pi}{3}$ b) $\frac{2\pi}{3}$ c) 2π d) 6π

Problem 30**1 point**

The base radius of a cone is twice less than the height of the cone. Find the ratio of the lateral area of the cone to the base area of the cone.

- a) $\sqrt{5}$ b) $\sqrt{3}$ c) $\frac{\sqrt{5}}{2}$ d) $2\sqrt{3}$

Problem 31**2 points**

Find the solution set of the system of inequalities

$$\begin{cases} 5x + 4 \geq 3x - 2 \\ x - 2 < -4x \end{cases} .$$

Problem 32**2 points**Find the sum of the first five terms of the geometric progression b_1, b_2, \dots if $b_1 = \frac{1}{2}$ and $b_4 = -4$.

Problem 33**2 points**

The diagonal of an isosceles trapezoid forms a 30° angle with a base. Find the altitude of the trapezoid if the midline of the trapezoid is equal to 8 cm.

Problem 34**2 points**

Solve the equation $\log_2(7x+5) = 4$.

Problem 35**3 points**

Find all the values of the parameter k for which the line $y = kx - 5$ has at least one common point with the parabola $y = 8x^2$.

Problem 36**3 points**

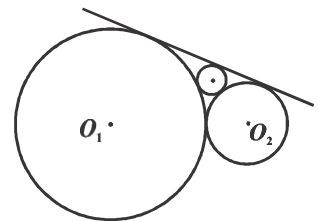
The fraction $\frac{m}{n}$, where m and n are positive integers is given. If we increase the numerator of the fraction by 5%, then by what percent should we increase the denominator in order to decrease the fraction $\frac{m}{n}$ by 10%?

Problem 37**3 points**

Points A and B lie on different bases of a cylinder. The plane α contains the segment AB and is parallel to the axis of the cylinder. Find the length of the segment AB if the angle between this segment and the plane of the base of the cylinder is 45° , the base radius of the cylinder is 5, and the distance between the axis of the cylinder and the plane α is 4.

Problem 38**4 points**

Two circles with centers at points O_1 and O_2 and of radii 3 and 1, respectively, touch each other externally. Between the circles and their common external tangent line the third circle is inscribed, which touches both given circles and their external tangent line as shown in the figure. Find the radius of the third circle.



Problem 39**4 points**

The company had to produce a certain number of products. After four hours of working, the main production line suddenly stopped, but a standby line started immediately. In order to produce the entire quantity, the standby line needs 6 hours more than the main production line. 3 hours after the accident, the main production line was repaired and it turned out that at that moment it had produced only half of the total number of the products. How long does it take to produce the remaining products, if both lines will continue to work together?

Problem 40**4 points**

The point $A(x, y)$ the coordinates of which at any time t satisfy the system of equations

$$\begin{cases} x = 8 \sin t + 6 \cos t \\ y = 4 \sin t - 12 \cos t \end{cases}$$

moves on the coordinate plane.

Find the minimal distance between the points $A(x, y)$ and $B(-1; 2)$ if $\frac{\pi}{2} \leq t \leq \frac{2\pi}{3}$.

Answers

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
b	b	c	d	a	b	d	d	b	d	c	c	b	a	d

16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
d	b	d	c	a	d	a	b	b	c	c	c	a	a	a

31	32	33	34	35	36
$-3 \leq x < \frac{2}{5}$	$\frac{11}{2}$	$\frac{8\sqrt{3}}{3}$	$\frac{11}{7}$	$(-\infty; -4\sqrt{10}] \cup [4\sqrt{10}; +\infty)$	$\frac{50}{3}\%$

37	38	39	40
$AB = h\sqrt{2}$, where h is the height of the cylinder and $0 < h \leq 6$. $AB = 6\sqrt{2}$ if the points A and B are located on the circles of the bases of the cylinder.	$\frac{3}{(\sqrt{3}+1)^2}$	$\frac{18}{5}h$	$\sqrt{76}$